Mathematical Models

• Compact Understanding of the World
Mathematical Models - Example

• Face Recognition with varying expressions

• Too Easy...
Mathematical Models

• Face Recognition with varying expressions
I. RANSAC

- Random Sample Consensus
- Used for Parametric Matching/Model Fitting
- Applications:
Line Fitting

• Fit the best possible Line to these points

• Brute Force Search – $2^N$ possibilities!!

• Not Feasible

• Better Strategy?
How RANSAC Works

• Random Search – Much Faster!!!
Line Fitting using RANSAC

• Iteration 1
Line Fitting using RANSAC

• Iteration 1
Line Fitting using RANSAC

• Iteration 1
Line Fitting using RANSAC

• Iteration 2
Line Fitting using RANSAC

• Iteration 2
Line Fitting using RANSAC

• Iteration 2
Line Fitting using RANSAC

- ...
- Iteration 5
Line Fitting using RANSAC

• Iteration 5
Line Fitting using RANSAC

• Iteration 5
Why RANSAC Works?

• Inliers vs Outliers

• \( P(\text{selecting outliers}) = \frac{17 \binom{2}{2}}{27 \binom{2}{2}} + \frac{17 \binom{1}{1} \cdot 10 \binom{1}{1}}{27 \binom{2}{2}} = 0.48 \)
Why RANSAC Works?

• Inliers vs Outliers

• $P(\text{selecting outliers}) = \frac{17C_2}{27C_2} + \frac{17C_1 \cdot 10C_1}{27C_2} = 0.48$

• After 5 iterations...

• $P(\text{selecting outliers}) = (0.48)^5 = 0.026$
Why RANSAC Works?

• In general:

\[ p = 1 - (1 - w^n)^k \]

Where,

p = probability for selecting inliers
w = ratio of inliers to total #points
n = minimum #points required (for line = 2, circle = 3)
k = #iterations
RANSAC Algorithms

Determine:
- $n$ — the smallest number of points required (e.g., for lines, $n = 2$, for circles, $n = 3$)
- $k$ — the number of iterations required
- $t$ — the threshold used to identify a point that fits well
- $d$ — the number of nearby points required
  to assert a model fits well

Until $k$ iterations have occurred
  Draw a sample of $n$ points from the data
    uniformly and at random
  Fit to that set of $n$ points
  For each data point outside the sample
    Test the distance from the point to the structure
      against $t$; if the distance from the point to the structure
      is less than $t$, the point is close
  end
  If there are $d$ or more points close to the structure
    then there is a good fit. Refit the structure using all
    these points. Add the result to a collection of good fits.
end
Use the best fit from this collection, using the
fitting error as a criterion

II. Least Squares

• Fitting Curves/Learning Data Manifolds
Line Fitting

• Goal: Find a line that best explains the observed data

• Target: $y_i$
• Data: $x_i$
• Line parameter: $w,b$
• Line Model:
  $y_i = w \cdot x_i + b$
Line Fitting

• Line Model:
  \[ y_i = w x_i + b \]

• Too many samples!

• Minimize error:
  \[ \min_{w,b} \sum_{i=1}^{N} (y_i - w x_i + b)^2 \]
Case 1 (m=n): Unique Solution

$$w = x \backslash y$$

No least square requires
# Samples (m) vs # Model-Parameters (n)

- Case 2 ($m > n$): Over-determined system of equations
- No Solution exists!
- Hence, we minimize error (fitting)
#Samples(m) $\text{vs}$ #Model-Parameters(n)

- Case 3 ($m<n$): Under-determined system of equations
- Infinite Solutions exist!
- Which one to choose?